STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

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IV.—NUMERICAL COMPUTATIONS IN THE VERTICAL ORDINATE.

THREE GENERAL THEORIES REGARDING THE FORMATION OF CYCLONES AND ANTICYCLONES.

There seem to be only three important general theories regarding the formation of cyclones and anticyclones in the earth's atmosphere, which may be referred to those authors who have been conspicuously associated with their mathematical developments: (1) Ferrel's cold center and warm center cyclones and anticyclones; (2) Oberbeck's symmetrical central cyclones and anticyclones; and (3) Bigelow's asymmetric cyclones and anticyclones. In my International Cloud Report, 1898, I reviewed the mathematical analyses of the first and second theories, and gave my reasons for thinking that they are inconsistent with the air currents as mapped out by the cloud observations, as well as with the distribution of temperature found in the lower strata. These theories start with the systems of isobars which near the surface are distributed symmetrically about a central axis, and they assume that the temperatures are similarly arranged, which is, however, not the case, as we know. The two central systems have their origin in the fact that the second equation of motion can be discussed in two ways. Thus, in the case of no friction, $k=0$, the equation

$$0=\frac{dv}{dt}+(2n\cos\theta+\nu_1)\,u+k\,v,$$

can be integrated by introducing the idea of a boundary cylinder about the system at the radial distance ϖ_0, whence is derived,

$$v=\left(\frac{2\varpi^2}{\varpi_0^2}-1\right)\varpi\, n\cos\theta,$$

which is the tangential velocity at the distance ϖ. This is Ferrel's method and several difficulties regarding it are mentioned on page 615 of the Cloud Report. The second equation of motion can be given another form retaining the friction term, where $\lambda=2n\cos\theta$, so that,

$$0=\frac{\partial v}{\partial t}+\frac{uv}{\varpi}+\lambda u+kv,$$

from which are derived two solutions,

$$\text{First}\begin{cases}u=-\dfrac{c}{2}\varpi\\[2ex] v=+\dfrac{\lambda}{k-c}\cdot\dfrac{c}{2}\varpi\end{cases},\qquad \text{Second}\begin{cases}u=-\dfrac{c}{\varpi}\\[2ex] v=+\dfrac{\lambda}{k}\dfrac{c}{\varpi}\end{cases}.$$

These form the basis of the theory developed by Guldberg and Mohn, Sprung, Oberbeck, Pockels, and others. My specific objections are summarized on page 623 of the Cloud Report.

The construction of a better theory was at that time very difficult, for two reasons, the first, that it involved breaking away from the large mass of current literature in meteorology, and that it introduced many new ideas concerning the general and the local circulations of the atmosphere, the two being intimately bound up together; the second, due to the lack of definite pressure and temperature observations in the higher strata of the atmosphere. In the course of chapters 8 and 11 of the Cloud Report the leading ideas regarding the asymmetric cyclone and anticyclone were sketched out, and a fairly clear idea was given of the probable truth regarding the formation of these circulating structures. Since that time the Weather Bureau has secured daily pressure maps for the United States on the three planes, sea level, 3,500 feet, and 10,000 feet, throughout an entire year, 1903. This valuable material has been carefully studied, and a report presented on the subject, with a summary of the results in the MONTHLY WEATHER REVIEW, May, 1904. The recent publications of the temperature observations, made during balloon and kite ascensions in Europe and America, have in some degree supplied this deficiency, and we are, therefore, now trying to discuss more definitely the entire subject by means of these several data—velocity, temperature, and pressure—than has been possible heretofore. In the preceding papers of this series we have given the temperature data and the thermodynamic formulæ, and in this paper we shall confine our attention to formula (44) in the vertical ordinate.

COMPARISON OF THE NUMERICAL RESULTS OF COMPUTATIONS BY FORMULA (38) AND THE GENERAL BAROMETRIC FORMULA OF THE CLOUD REPORT (59).

Since we have introduced a new system of formulæ for the computation of the pressures, densities, and the gas factors, from the temperatures, through the use of the ratio n, the ratio of the adiabatic temperature gradient to the observed gradient, it will be desirable to compare the numerical results by some examples, showing the relation of the thermodynamic formulæ to the well-known barometric formulæ. Formula (59) can be written as follows:

$$z-z_0=18400\left(\frac{273+\theta}{273}\right)\left(\frac{B+.378\,e}{B}\right)\frac{g_0}{g}\left(1+\frac{1.25\,K\,M}{R}\right)\log\frac{B_0}{B}$$

In our new thermodynamic formula we have made no attempt to refine it by introducing corrections due to the vapor term, $\dfrac{B+.378\,e}{B}$, the gravity term, $\dfrac{g_0}{g}$, nor the land-mass term, $\left(1+\dfrac{1.25\,K\,M}{R}\right)$. It is evident that these will require a small change in the value of n, and the subject is worth an investigation, but for our preliminary studies of cyclones and anticyclones these refinements have been omitted. We retain, then, simply

$$z-z_0=18400\ (1+.00367\ \theta)\ \log\frac{B_0}{B}\,\ldots\ldots\ldots\text{(II)}$$

where $\theta=\dfrac{T-T_0}{2}-273°$, the mean departure of the temperature of the air column from zero centigrade. This is to be compared with the formula,

$$-\,n\,\frac{k}{k-1}\,(\log T-\log T_0)=\log\frac{P_0}{P}=\log\frac{B_0}{B},\ \ldots\ldots\text{(I)}$$

and it will be sufficient to show that

$$\frac{z-z_0}{18400+67.5\,\theta}=-\,n\,\frac{k}{k-1}\,(\log T-\log T_0).$$

The examples are taken at random with sufficient range to test the formulæ severely. The observed gradient is found from $a=\dfrac{T-T_0}{z-z_0}$, per 1000 meters. The computation is given in full as an example of the numerical quantities involved. While the agreement in the logarithms is not perfect, the differences I—II are small for so great ranges of temperature and height when translated into millimeters of mercury. If $B_0=760.00$ mm. in the fifth example, for the difference 0.00084 the value of B is 294.00 and 294.57, respectively. As my only purpose is to illustrate the numerical validity of the n formula, it will not be necessary to inquire further into the causes of the small differences between I and II.

COMPUTATION OF MEAN VALUES OF P_0, ρ_0, R_0, FROM T_0 ON THE 1000-METER LEVELS.

In making an application of the formula (44),

$$\frac{P_0}{\rho_0}-\frac{P}{\rho}=\tfrac{1}{2}\,(q^2-q_0^2)+g\,(z-z_0)=Q-Q_0-C_p\,n\,(T-T_0)$$
$$-\,C_p\,T\log T\,(n-n_0),$$

TABLE 15.—*Comparison of the formulæ.*

	Formula I $= -n\frac{k}{k-1}$ ($\log T - \log T_0$)								
T	271.5	253.5	259.0	219.6	233.2	290.0	300.0	275.0	280.0
T_0	275.8	275.8	268.7	253.5	271.5	273.0	280.0	300.0	310.0
$T-T_0$	— 4.3	—22.3	— 9.7	—33.9	—38.3	+17.0	+20.0	—25.0	—30.0
Adiabatic.	—9.8695								
Observed.	—4.30	—4.46	—4.85	—6.78	—5.47	+4.25	+4.00	—5.00	—5.00
— log n	—0.36082	—0.34496	—0.30855	—0.16306	—0.25630	0.36590	0.39223	—0.29532	—0.29532
$k/k-1$	0.53927								
log T	2.43377	2.40398	2.41330	2.34163	2.36773	2.46240	2.47712	2.43933	2.44716
log T_0	2.44059	2.44059	2.42927	2.40398	2.43377	2.43616	2.44716	2.47712	2.49136
log T — log T_0	—0.00682	—0.03661	—0.01596	—0.06235	—0.06604	0.02624	0.02996	—0.03779	—0.04420
log (log T — log T_0)	—7.83378	—8.56360	—8.20330	—8.79484	—8.81981	8.41896	8.47654	—8.57738	—8.64542
log I	8.73387	9.44783	9.05112	9.49717	9.61538	9.32413	9.40804	9.41197	9.48001
I	0.05418	0.28043	0.11249	0.31417	0.41246	0.21092	0.25588	0.25821	0.30200
	Formula II $= \frac{z-z_0}{18400+67.5\,\theta}$								
$\frac{T+T_0}{2}$	273.65	264.65	263.85	236.55	252.35	281.5	290.0	287.5	295.0
θ	+0.65	—8.35	—9.15	—36.45	—20.65	+ 8.5	+17.0	+14.5	+22.0
$z-z_0$	1000	5000	2000	5000	7000	4000	5000	5000	6000
67.5 θ	+ 44	— 564	— 618	— 2460	— 1394	+ 574	+1148	+ 979	+1485
K	18400								
K + 67.5 θ	18444	17836	17782	15940	17006	18974	19548	19379	19885
II	0.05422	0.28034	0.11247	0.31368	0.41162	0.21082	0.25578	0.25801	0.30173
I — II	—.00004	+.00009	+.00002	+.00049	+.00084	+.00010	+.00010	+.00020	+.00027

to the earth's atmosphere, it is evident that we must first compute the values of P_0, ρ_0, R_0, corresponding to T_0 as observed in the air while it is undisturbed by the local cyclonic and anticyclonic circulations. The observations of temperature were actually made in the midst of the prevailing local disturbances, but the average temperature of the air on the 1000-meter levels was found by taking the mean temperatures of the eight sectors, four in the high areas and four in the low areas, as in Tables 10 and 11. (See MONTHLY WEATHER REVIEW, February, 1906, Vol. XXXIV, p. 75.) Thus, Table 11 gives the adopted mean temperatures and gradients on the 1000-meter levels for American and European cyclones and anticyclones, and these data are used in the following computations.

The values of n are found on dividing 9.8695 by the gradient per 1000 meters, assuming that the gradient is a constant between the two levels. In a later section we shall compute, also, the values of n on the 1000-meter levels themselves, taking as the gradients those found on fig. 8 at the points indicated. We use the mean gradient between two levels in computing P_0, ρ_0, R_0, and then the gradient at a given level to compute P, ρ, R, the abnormal values of pressure, density, and gas factor produced by the local cyclonic and anticyclonic disturbances. Having found n from level to level the following formulæ are applied in succession, including 41 as a check:

$$\log P = \log P_0 + n\,\frac{k}{k-1}\,(\log T - \log T_0). \tag{38}$$

$$\log \rho = \log \rho_0 + \frac{n}{k-1}\,(\log T - \log T_0). \tag{39}$$

$$\log R = \log R_0 + (n-1)\,(\log T - \log T_0). \tag{40}$$

$$\log \rho = \log \rho_0 + \frac{1}{k}\,(\log P - \log P_0). \tag{41}$$

A special point should be noted in connection with the symbols. P_0 is the pressure on one level, as the 1000-meter level, and then P is the pressure on another level, as the 2000-meter level, corresponding with T_0 and T, respectively. In this way a succession of values of P is found in the several strata of the undisturbed atmosphere, applying to the general circulation only. In studying the pressure variations in cyclones and anticyclones these P-pressures become P_0 in formula (44), the P-values of that formula referring to disturbances within the local circulation on a given level. I have preferred to make this explanation rather than complicate the formulæ with additional symbols for all contingencies. In computing the tables following, the pressure $P_0 = 101323$, for $B = 760$ mm., was taken as the initial value. The initial value of the density on the sea-level plane was computed from,

$$\rho_0 = \frac{P_0}{R_0\,T_0},$$

where $R_0 = 287.0334$, and T_0 is the value from Table 11, $T_0 =$ 275.8, 277.3, 290.2, 287.1. While it is true that the atmosphere is seldom in the state represented by these tables, yet it fluctuates about these mean values, just as it does about the mean pressure, temperature, and density at sea level, and it is convenient to have reference values from which to conduct our discussions.

TABLE 16.—*Computed values of the ratio n between successive 1000-meter levels.*

Height in meters.	American.		European.	
	Winter.	Summer.	Winter.	Summer.
16000				
	3.037	2.820	3.037	3.589
14000				
	4.386	2.820	4.386	3.037
12000				
	2.078	1.716	2.078	1.645
10000				
	1.518	1.473	1.518	1.410
9000				
	1.390	1.410	1.410	1.316
8000				
	1.410	1.518	1.410	1.410
7000				
	1.451	1.673	1.410	1.518
6000				
	1.518	1.518	1.410	1.618
5000				
	1.794	1.410	1.673	1.702
4000				
	1.974	1.316	1.794	1.935
3000				
	2.100	1.653	2.014	2.295
2000				
	3.525	1.828	2.243	1.673
1000				
	2.295	1.828	2.467	1.645
000				

Since $n = \frac{-9.8695}{T-T_0}$, the variation of n in Table 16 is a function of ΔT. Where ΔT is large, n is small, and inversely. Hence, in the lower and the higher levels n is larger than in the middle levels, the change of temperature being slower below and above, for the reasons already given. In the middle levels $n = 1.5$ approximately, and it may become twice as great in higher or lower strata. Tables 17, 18, 19, and 20 contain the values of P_0, ρ_0, R_0, T_0 on the several levels, and their logarithms which are useful in computations. Since $P_0 = g_0 \rho_m B_m$, we have

$$B_n = \frac{P_0}{g_0 \rho_m} = \frac{P_0}{9.806 \times 13595.8} = \frac{P_0}{133330} = \frac{P_0}{100} \times \frac{3}{4}, \text{ in millimeters.}$$

The pressure is higher in summer than in winter on the same level; the density is higher in winter than in summer; the gas factor is higher in summer than in winter; and the temperature is higher in summer than in winter, the difference diminishing in the upper levels. The gas factor is not a constant in any system except the adiabatic, where $n = 1$.

COLLECTION OF THE DATA SHOWING THE DISTRIBUTION OF THE DISTURBANCES ON THE 1000-METER LEVELS.

We will collect the data in a form suitable for the discussion of the distribution of the energy in cyclones and anticyclones, leaving the reader to make his own inferences by an examination of the tables in their relation to one another.

Computed mean values of the pressure, density, and gas factor from the temperature at several elevations.

TABLE 17.—EUROPEAN WINTER.

Height in meters.	P_0	log P_0	ρ_0	log ρ_0	R_0	log R_0	T_0	log T_0
16000......	9547	3.97987	0.23733	9.37535	203.59	2.30874	197.6	2.29579
14000......	13414	4.12756	0.30226	9.48038	217.45	2.33736	204.1	2.30984
12000......	18679	4.27135	0.38250	9.58263	234.12	2.36943	208.6	2.31931
10000......	25735	4.41052	0.48040	9.68160	245.63	2.39029	218.1	2.33866
9000......	30097	4.47753	0.53610	9.72925	249.40	2.39688	224.6	2.35141
8000......	34881	4.54259	0.59636	9.77551	252.55	2.40235	231.6	2.36474
7000......	40335	4.60569	0.66127	9.82038	255.65	2.40765	238.6	2.37767
6000......	46450	4.66699	0.73108	9.86397	258.70	2.41280	245.6	2.39023
5000......	53276	4.72653	0.80595	9.90631	261.70	2.41780	252.6	2.40243
4000......	60897	4.78461	0.88636	9.94761	265.80	2.42455	258.5	2.41246
3000......	69403	4.84138	0.99536	9.98798	270.28	2.43181	264.0	2.42160
2000	78902	4.89709	1.06559	0.02759	275.37	2.43991	268.9	2.42959
1000......	89502	4.95183	1.16551	0.06652	280.98	2.44867	273.3	2.43664
000......	101323	5.00571	1.27300	0.10483	287.03	2.45793	277.3	2.44295

TABLE 18.—EUROPEAN SUMMER.

Height in meters.	P_0	log P_0	ρ_0	log ρ_0	R_0	log R_0	T_0	log T_0
16000......	10187	4.00804	0.24003	9.38027	210.20	2.32262	201.9	2.30514
14000......	14224	4.15302	0.30434	9.48336	225.34	2.35283	207.4	2.31681
12000......	19674	4.29388	0.38329	9.58353	239.96	2.38013	213.9	2.33021
10000......	26816	4.42888	0.47811	9.67953	248.55	2.39542	225.9	2.35392
9000......	31156	4.49355	0.53152	9.72552	251.68	2.40085	232.9	2.36717
8000......	35994	4.55623	0.58896	9.77009	254.21	2.40520	240.4	2.38093
7000......	41408	4.61709	0.65069	9.81337	257.22	2.41031	247.4	2.39340
6000......	47453	4.67627	0.71690	9.85546	260.70	2.41614	253.9	2.40466
5000......	54201	4.73401	0.78802	9.89654	264.55	2.42241	260.0	2.41497
4000......	61730	4.79050	0.86440	9.93671	268.68	2.42924	265.8	2.42455
3000......	70118	4.84583	0.94638	9.97606	273.50	2.43696	270.9	3.43281
2000......	79464	4.90017	1.03443	0.01470	279.14	2.44582	275.2	2.43965
1000......	89846	4.95350	1.12882	0.05262	283.15	2.45202	281.1	2.44886
000......	101323	5.00571	1.22956	0.08975	287.03	2.45793	287.1	2.45803

Computed mean values of the pressure, density, and gas factor from the temperature at several elevations.

TABLE 19.—AMERICAN WINTER.

Height in meters.	P_0	log P_0	ρ_0	log ρ_0	R_0	log R_0	T_0	log T_0
16000......	9651	3.98456	0.24045	9.38103	201.59	2.30446	199.1	2.29907
14000......	13527	4.13120	0.30571	9.48531	215.22	2.33289	205.6	2.31302
12000......	18797	4.27408	0.38630	9.58691	231.60	2.36497	210.1	2.32243
10000......	25833	4.41217	0.48430	9.68511	242.91	2.38544	219.6	2.34163
9000.....	30114	4.47876	0.54010	9.73247	246.61	2.39200	226.1	2.35430
8000......	34945	4.54338	0.60037	9.77842	249.60	2.39724	233.2	2.36773
7000......	40369	4.60605	0.66524	9.82298	252.64	2.40250	240.2	2.38057
6000......	46450	4.66699	9.73505	9.86632	255.84	2.40797	247.0	2.39270
5000......	53251	4.72633	0.81006	9.90852	259.31	2.41382	253.5	2.40399
4000......	60836	4.78416	0.89052	9.94964	263.76	2.42121	259.0	2.41330
3000......	69321	4.84087	0.97718	9.98997	268.71	2.42929	264.0	2.42160
2000......	78817	4.89662	1.07059	0.02962	273.99	2.43773	268.7	2.42927
1000......	89440	4.95153	1.17127	0.06866	281.25	2.44910	271.5	2.43377
000.... .	101323	5.00571	1.27994	0.10719	287.03	2.45793	275.8	2.44059

TABLE 20.—AMERICAN SUMMER.

Height in meters.	P_0	log P_0	ρ_0	log ρ_0	R_0	log R_0	T_0	log T_0
16000......	10232	4.00995	0.23822	9.37698	213.88	2.33017	200.8	2.30276
14000......	14298	4.15529	0.30223	9.48033	227.65	2.35727	207.8	2.31765
12000	19754	4.29565	0.38031	9.58014	241.79	2.38344	214.8	2.33203
10000......	26929	4.43022	0.47407	9.67584	250.99	2.39966	226.3	2.35468
9000......	31252	4.49488	0.52701	9.72182	254.48	2.40566	233.0	2.36736
8000......	36107	4.55759	0.58401	9.76642	257.59	2.41093	240.0	2.38021
7500......	41554	4.61861	0.64537	9.80981	261.18	2.41694	246.5	2.39182
6000......	47652	4.67808	0.71138	9.85210	265.37	2.42385	252.4	2.40209
5000	54449	4.73599	0.78213	9.89328	268.88	2.42956	258.9	2.41313
4000......	62023	4.79255	0.85802	9.93350	271.84	2.43431	265.9	2.42472
3000......	70402	4.84758	0.93892	9.97263	274.24	2.43813	273.4	2.43680
2000......	79712	4.90152	1.02563	0.01099	278.16	2.44429	279.4	2.44623
1000	89968	4.95409	1.11782	0.04837	282.60	2.45117	284.8	2.45454
000......	101323	5.00571	1.21642	0.08508	287.03	2.45793	290.2	2.46270

VALUES OF THE TEMPERATURES T, T_0 AND $T - T_0$.

The temperature data given in Tables 12 and 13 (see MONTHLY WEATHER REVIEW, February, 1906, Vol. XXXIV, pp. 76, 77) have been reproduced in Tables 21, 22, 23, and 24, in a form more convenient for carrying on the computations depending upon them. T is the temperature in the several sectors; T_0 is the mean temperature on the same level computed from the eight sectors of the correlative high and low areas; $T - T_0$ is the departure of the disturbed area from the assumed mean undisturbed atmosphere without any cyclonic and anticyclonic action, and is shown on figs. 5 and 6.

VALUES OF THE RATIOS n, n_0 AND $(n - n_0)$.

These are the ratios on the several 1000-meter levels, and they are used for computing the energy of the local disturbances on a given plane, rather than in reducing the elements from one plane to another, as was required for constructing Tables 17, 18, 19, and 20. The values of n on the several levels for each sector were scaled from fig. 8, for which purpose it was constructed, and from them the values of n in Tables 25, 26, 27, and 28 were computed. The values of n_0, the mean ratio, were found by taking the means of the eight values of n in the sectors on the given plane. Tables 25, 26, 27, and 28 contain the data n, n_0, and the differences $n - n_0$, the distribution of $n - n_0$ being given in fig. 14 on the several levels. Since $n_0 = \frac{-9.8695}{T - T_0}$ is a certain average gradient, if n

Values of T, T_0, $T-T_0$ derived from Tables 12, 13.

TABLE 21.—WINTER HIGH AREAS.

Height in meters.	T N.	T E.	T S.	T W.	T_0 Mean.	$T-T_0$ N.	$T-T_0$ E.	$T-T_0$ S.	$T-T_0$ W.
	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.
10000	−55.2	−56.2	−53.7	−52.2	−54.2	−1.0	−2.0	+0.5	+2.0
9000	−48.2	−50.2	−46.8	−45.3	−47.7	−0.5	−2.5	+0.9	+2.4
8000	−40.6	−43.6	−39.3	−37.6	−40.6	0.0	−3.0	+1.3	+3.0
7000	−32.9	−37.1	−32.1	−29.6	−33.6	+0.7	−3.5	+1.5	+4.0
6000	−25.3	−30.7	−25.5	−21.7	−26.7	+1.4	−4.0	+1.2	+5.0
5000	−18.0	−24.6	−20.0	−14.3	−20.0	+2.0	−4.6	0.0	+5.7
4000	−11.5	−18.6	−15.5	− 8.7	−14.3	+2.8	−4.3	−1.2	+5.6
3000	− 6.8	−13.0	−10.3	− 4.2	− 9.0	+2.7	−4.0	−1.3	+4.8
2000	− 2.2	− 8.0	− 6.9	− 0.2	− 4.2	+2.0	−3.8	−2.7	+4.0
1000	+ 0.9	− 3.7	− 3.1	+ 2.7	− 0.6	+1.5	−3.1	−2.5	+3.3
000	+ 4.2	+ 1.5	+ 0.1	+ 5.6	+ 3.6	+0.6	−2.1	−3.5	+2.0

TABLE 22.—WINTER LOW AREAS.

Height in meters.	T N.	T E.	T S.	T W.	T_0 Mean.	$T-T_0$ N.	$T-T_0$ E.	$T-T_0$ S.	$T-T_0$ W.
10000	−52.0	−51.2	−56.2	−57.0	−54.2	+2.2	+3.0	−2.0	−2.8
9000	−45.7	−44.7	−49.9	−50.7	−47.7	+2.0	+3.0	−2.2	−3.0
8000	−39.1	−37.9	−43.1	−43.9	−40.6	+1.5	+2.7	−2.5	−3.3
7000	−32.4	−31.1	−36.2	−37.1	−33.6	+1.2	+2.5	−2.6	−3.5
6000	−25.6	−24.5	−29.5	−30.5	−26.7	+1.1	+2.2	−2.8	−3.8
5000	−18.8	−18.0	−22.8	−24.0	−20.0	+1.2	+2.0	−2.8	−4.0
4000	−12.5	−12.5	−16.7	−17.8	−14.3	+1.8	+1.8	−2.4	−3.5
3000	− 6.8	− 7.6	−11.3	−12.4	− 9.0	+2.2	+1.4	−2.3	−3.4
2000	− 1.8	− 3.0	− 5.5	− 7.2	− 4.2	+2.4	+1.2	−1.3	−3.0
1000	+ 1.1	+ 0.7	+ 0.5	− 2.8	− 0.6	+1.7	+1.3	+1.1	−2.2
000	+ 4.5	+ 4.7	+ 6.5	+ 1.9	+ 3.6	+0.9	+1.1	+2.9	−1.7

TABLE 23.—SUMMER HIGH AREAS.

Height in meters.	T N.	T E.	T S.	T W.	T_0 Mean.	$T-T_0$ N.	$T-T_0$ E.	$T-T_0$ S.	$T-T_0$ W.
10000	−48.1	−48.4	−46.7	−45.9	−46.9	−1.2	−1.5	+0.2	+1.0
9000	−40.7	−42.1	−39.6	−38.6	−40.1	−0.6	−2.0	+0.5	+1.5
8000	−32.8	−35.3	−31.8	−31.0	−32.8	0.0	−2.5	+1.0	+1.8
7000	−25.4	−29.1	−24.6	−24.1	−26.1	+0.7	−3.0	+1.5	+2.0
6000	−18.5	−23.2	−18.7	−17.5	−19.9	+1.4	−3.3	+1.2	+2.4
5000	−11.7	−16.8	−13.7	−10.8	−13.6	+1.9	−3.2	−0.1	+2.8
4000	− 4.9	−10.4	− 7.7	− 5.0	− 7.2	+2.3	−3.2	−0.5	+2.2
3000	+ 1.0	− 3.8	− 1.3	+ 1.1	− 0.9	+1.9	−2.9	−0.4	+2.0
2000	+ 5.7	+ 2.1	+ 3.9	+ 5.9	+ 4.3	+1.4	−2.2	−0.4	+1.6
1000	+10.5	+ 8.1	+ 8.8	+11.0	+10.0	+0.5	−1.9	−1.2	+1.0
000	+15.9	+14.1	+13.7	+16.1	+15.7	+0.2	−1.6	−2.0	+0.4

TABLE 24.—SUMMER LOW AREAS.

Height in meters.	T N.	T E.	T S.	T W.	T_0 Mean.	$T-T_0$ N.	$T-T_0$ E.	$T-T_0$ S.	$T-T_0$ W.
10000	−42.1	−45.7	−48.9	−49.4	−46.9	+4.8	+1.2	−2.0	−2.5
9000	−35.5	−38.9	−42.4	−42.9	−40.1	+4.6	+1.2	−2.3	−2.8
8000	−28.4	−31.8	−35.5	−35.8	−32.8	+4.4	+1.0	−2.7	−3.0
7000	−21.9	−24.9	−29.1	−29.3	−26.1	+4.2	+1.2	−3.0	−3.2
6000	−15.9	−18.5	−22.9	−23.4	−19.9	+4.0	+1.4	−3.0	−3.5
5000	−10.3	−12.2	−16.3	−16.6	−13.6	+3.3	+1.4	−2.7	−3.0
4000	− 4.4	− 5.9	− 9.2	− 9.8	− 7.2	+2.8	+1.3	−2.0	−2.6
3000	+ 1.5	− 0.6	− 1.9	− 3.5	− 0.9	+2.4	+0.3	−1.0	−2.6
2000	+ 6.2	+ 4.5	+ 4.3	+ 1.9	+ 4.3	+1.9	+0.2	0.0	−2.4
1000	+11.2	+10.4	+11.0	+ 7.8	+10.0	+1.2	+0.4	+1.0	−2.2
000	+16.5	+16.5	+18.5	+14.2	+15.7	+0.8	+0.8	+2.8	−1.5

Distribution of the values of n, n_0, $n-n_0$.

TABLE 25.—WINTER HIGH AREAS.

Height in meters.	n N.	n E.	n S.	n W.	n_0 Mean.	$n-n_0$ N.	$n-n_0$ E.	$n-n_0$ S.	$n-n_0$ W.
10000	1.507	1.778	1.574	1.535	1.600	−.093	+.178	−.026	−.065
9000	1.339	1.537	1.367	1.319	1.444	−.105	+.093	−.077	−.125
8000	1.282	1.495	1.348	1.246	1.396	−.114	+.099	−.048	−.150
7000	1.283	1.523	1.404	1.234	1.403	−.120	+.120	+.001	−.169
6000	1.323	1.567	1.613	1.272	1.459	−.136	+.108	+.154	−.187
5000	1.420	1.623	2.100	1.430	1.602	−.182	+.021	+.498	−.172
4000	1.747	1.725	2.317	1.974	1.819	−.072	−.094	+.498	+.155
3000	2.274	1.894	2.804	2.443	2.105	+.169	−.211	+.699	+.338
2000	2.920	2.285	2.937	3.056	2.554	+.366	−.269	+.383	+.502
1000	3.184	2.179	2.903	3.439	2.661	+.523	−.482	+.242	+.778
000	2.991	1.769	2.467	3.290	2.403	+.588	−.634	+.064	+.887

TABLE 26.—WINTER LOW AREAS.

Height in meters.	n N.	n E.	n S.	n W.	n_0 Mean.	$n-n_0$ N.	$n-n_0$ E.	$n-n_0$ S.	$n-n_0$ W.
10000	1.587	1.547	1.613	1.659	1.600	−.013	−.053	+ .013	+.059
9000	1.528	1.458	1.500	1.500	1.444	+.084	+.014	+ .056	+.056
8000	1.471	1.447	1.437	1.443	1.396	+.075	+.051	+ .041	+.047
7000	1.430	1.471	1.430	1.447	1.403	+.027	+.068	+ .027	+.044
6000	1.426	1.518	1.458	1.498	1.459	−.033	+.059	− .001	+.039
5000	1.489	1.629	1.564	1.562	1.602	−.113	+.027	− .038	−.040
4000	1.629	1.766	1.707	1.690	1.819	−.190	−.053	− .112	−.129
3000	1.848	1.974	1.725	1.876	2.105	−.257	−.131	− .380	−.229
2000	2.903	2.518	1.667	2.150	2.554	+.347	−.036	− .887	−.404
1000	3.123	2.611	1.637	2.213	2.661	+.462	−.050	−1.024	−.448
000	2.632	2.367	1.645	2.065	2.403	+.229	−.036	− .758	−.338

TABLE 27.—SUMMER HIGH AREAS.

Height in meters.	n N.	n E.	n S.	n W.	n_0 Mean.	$n-n_0$ N.	$n-n_0$ E.	$n-n_0$ S.	$n-n_0$ W.
10000	1.371	1.623	1.430	1.386	1.503	−.132	+.120	−.073	−.117
9000	1.233	1.482	1.307	1.328	1.382	−.149	+.100	−.075	−.054
8000	1.265	1.491	1.309	1.360	1.396	−.131	+.095	−.087	−.036
7000	1.392	1.662	1.537	1.469	1.539	−.147	+.123	−.002	−.070
6000	1.443	1.667	1.873	1.673	1.628	−.185	+.039	+.245	+.045
5000	1.447	1.589	1.905	1.725	1.607	−.160	−.018	+.298	+.118
4000	1.542	1.505	1.577	1.631	1.563	−.021	−.058	+.014	+.068
3000	1.947	1.569	1.645	1.818	1.728	+.219	−.159	−.083	+.090
2000	2.113	1.681	1.909	1.974	1.869	+.244	−.188	+.040	+.105
1000	1.905	1.637	1.974	1.909	1.753	+.152	−.116	+.221	+.156
000	1.785	1.645	2.014	1.902	1.688	+.097	−.043	+.326	+.214

TABLE 28.—SUMMER LOW AREAS.

Height in meters.	n N.	n E.	n S.	n W.	n_0 Mean.	$n-n_0$ N.	$n-n_0$ E.	$n-n_0$ S.	$n-n_0$ W.
10000	1.607	1.500	1.572	1.535	1.503	+.104	−.003	+.069	+.032
9000	1.443	1.394	1.445	1.422	1.382	+.061	+.012	+.063	+.040
8000	1.439	1.398	1.478	1.426	1.396	+.043	+.002	+.082	+.030
7000	1.559	1.516	1.597	1.582	1.539	+.020	−.023	+.058	+.043
6000	1.725	1.574	1.564	1.507	1.628	+.097	−.054	−.064	−.121
5000	1.744	1.569	1.430	1.447	1.607	+.137	−.038	−.177	−.160
4000	1.684	1.719	1.352	1.491	1.563	+.121	+.156	−.211	−.072
3000	1.769	1.943	1.386	1.744	1.728	+.041	+.215	−.342	+.016
2000	2.109	1.855	1.557	1.750	1.869	+.440	−.014	−.312	−.119
1000	1.974	1.673	1.384	1.572	1.753	+.221	−.080	−.369	−.181
000	1.781	1.597	1.279	1.502	1.688	+.093	−.091	−.409	−.186

Distribution of the heights $z - z_0$. $(z - z_0)\ g = -\ C_p\ n_0\ (T - T_0.)$

TABLE 29.—WINTER HIGH AREAS.

Height in meters.	$-C_p n_0 (T-T_0)$ N.	E.	S.	W.	g	$z-z_0$ N.	E.	S.	W.
10000	1590	3180	— 795	—3180	9.786	+162	+325	— 81	— 325
9000	717	3587	—1291	—3443	9.789	+ 73	+366	—132	— 352
8000	0	4161	—1803	—4161	9.791	0	+425	—184	— 425
7000	— 976	4879	—2091	—5576	9.793	—100	+498	—213	— 569
6000	—2030	5799	—1739	—7248	9.795	—207	+592	—178	— 740
5000	—3183	7322	0	—9075	9.796	—325	+748	0	— 926
4000	—5060	7772	2169	—10121	9.798	—516	+793	+221	—1033
3000	—5647	8366	2719	—10039	9.800	—576	+854	+278	—1024
2000	—5075	9643	6851	—10150	9.802	—518	+984	+699	—1036
1000	—3966	8196	6610	—8725	9.804	—405	+836	+674	— 890
000	—1433	5014	8356	—4775	9.806	—146	+511	+853	— 487

TABLE 30.—WINTER LOW AREAS.

Height in meters.	$-C_p n_0 (T-T_0)$ N.	E.	S.	W.	g	$z-z_0$ N.	E.	S.	W.
10000	—3497	—4769	3180	4451	9.786	—357	—487	+325	+454
9000	—2869	—4304	3156	4304	9.789	—280	—440	+322	+440
8000	—2081	—3745	3468	4577	9.791	—213	—382	+354	+467
7000	—1673	—3485	3624	4879	9.793	—171	—356	+370	+498
6000	—1595	—3189	4059	5509	9.795	—163	—326	+414	+563
5000	—1910	—3183	4457	6367	9.796	—195	—325	+455	+650
4000	—3253	—3253	4338	6326	9.798	—332	—332	+443	+646
3000	—4601	—2928	4810	7111	9.800	—591	—299	+491	+726
2000	—6090	—3045	3299	7613	9.802	—621	—311	+337	+777
1000	—4495	—3437	—2908	5817	9.804	—459	—351	—297	+593
000	—2149	—2626	—6924	4059	9.806	—219	—268	—706	+414

TABLE 31.—SUMMER HIGH AREAS.

Height in meters.	$-C_p n_0 (T-T_0)$ N.	E.	S.	W.	g	$z-z_0$ N.	E.	S.	W.
10000	1794	2240	— 299	—1492	9.786	+183	+229	— 31	—153
9000	824	2746	— 687	—2060	9.789	+ 84	+280	— 70	—211
8000	0	3468	—1387	—2497	9.791	0	+354	—142	—255
7000	—1071	4587	—2294	—3058	9.793	—109	+468	—234	—312
6000	—2265	5338	—1941	—3882	9.795	—231	+545	—198	—397
5000	—3034	5109	160	—4471	9.796	— 312	+522	+ 16	—457
4000	—3572	4970	777	—3416	9.798	—364	+507	+ 79	—349
3000	—3262	4979	687	—3434	9.800	—333	+508	+ 70	—350
2000	—2600	4085	743	—2971	9.802	—265	+417	+ 76	—303
1000	— 871	3309	2090	—1742	9.804	— 88	+338	+213	—178
000	— 669	2683	3354	— 671	9.806	— 68	+274	+342	— 68

TABLE 32.—SUMMER LOW AREAS.

Height in meters.	$-C_p n_0 (T-T_0)$ N.	E.	S.	W.	g	$z-z_0$ N.	E.	S.	W.
10000	—7168	—1792	2987	3733	9.786	—732	—183	+305	+381
9000	—6316	—1648	3158	3845	9.789	—645	—168	+322	+393
8000	—6103	—1387	3745	4161	9.791	—623	—142	+382	+425
7000	—6422	—1835	4587	4893	9.793	—656	—169	+468	+500
6000	—6470	—2265	4864	5661	9.795	—661	—231	+496	+577
5000	—5269	—2236	4311	4790	9.796	—538	—228	+440	+489
4000	—4348	—2019	3106	4038	9.798	—444	—207	+317	+412
3000	—4120	— 515	1717	4464	9.800	—420	— 53	+175	+455
2000	—3528	— 371	0	4457	9.802	—360	— 38	0	+454
1000	—2090	— 697	—1742	3832	9.804	—213	— 71	—178	+390
000	—1342	—1342	—4696	2481	9.806	—137	—137	—479	+253

Distribution of the velocities q, q_0, $\frac{1}{2}(q^2 - q_0^2)$.

TABLE 33.—WINTER HIGH AREAS.

Height in meters.	q N.	E.	S.	W.	q_0 Mean.	$\frac{1}{2}(q^2-q_0^2)$ N.	E.	S.	W.
10000	44	39	31	34	37.6	+261	+54	—227	—127
9000	43	38	28	33	36.5	+259	+56	—274	—122
8000	41	37	26	32	35.0	+228	+72	—275	—101
7000	39	35	23	30	32.9	+220	+72	—277	— 91
6000	37	32	20	27	29.9	+238	+65	—247	— 83
5000	34	30	17	24	26.9	+216	+88	—218	— 74
4000	32	28	16	23	25.3	+192	+72	—192	— 56
3000	30	26	14	21	23.6	+172	+60	—181	— 58
2000	25	20	12	18	19.5	+123	+10	—118	— 28
1000	19	15	10	14	14.8	+ 71	+ 3	— 60	— 12
000	9	9	6	7	8.0	+ 8	+ 8	— 14	— 8

TABLE 34.—WINTER LOW AREAS.

Height in meters.	q N.	E.	S.	W.	q_0 Mean.	$\frac{1}{2}(q^2-q_0^2)$ N.	E.	S.	W.
10000	31	35	45	42	37.6	—227	— 95	+306	+175
9000	30	33	45	42	36.5	—216	—122	+347	+216
8000	28	31	44	41	35.0	—221	—132	+356	+228
7000	26	29	42	39	32.9	—203	—121	+342	+220
6000	23	27	38	35	29.9	—183	— 83	+275	+166
5000	21	25	34	30	26.9	—142	— 50	+216	+ 88
4000	19	24	32	28	25.3	—140	— 32	+192	+ 72
3000	18	23	30	27	23.6	—117	— 14	+172	+ 86
2000	15	18	26	22	19.5	— 78	— 28	+148	+ 52
1000	12	13	20	16	14.8	— 38	— 25	+ 96	+ 19
000	6	7	10	10	8.0	— 16	— 8	+ 18	+ 18

TABLE 35.—SUMMER HIGH AREAS.

Height in meters.	q N.	E.	S.	W.	q_0 Mean.	$\frac{1}{2}(q^2-q_0^2)$ N.	E.	S.	W.
10000	37	33	26	29	33.5	+124	—12	—223	—141
9000	35	32	24	28	32.5	+120	—16	—240	—136
8000	35	31	22	27	31.1	+129	— 3	—242	—119
7000	33	29	19	25	29.0	+124	0	—240	—108
6000	31	27	17	23	26.5	+130	+14	—207	— 87
5000	29	25	14	20	23.9	+135	+27	—188	— 86
4000	27	24	13	19	22.4	+114	+37	—166	— 71
3000	25	22	12	18	21.0	+ 92	+22	—124	— 59
2000	21	17	10	15	17.3	+ 71	— 5	—100	— 37
1000	16	12	8	12	13.1	+ 42	—14	— 54	— 14
000	8	7	5	6	7.3	+ 6	— 2	— 14	— 9

TABLE 36.—SUMMER LOW AREAS.

Height in meters.	q N.	E.	S.	W.	q_0 Mean.	$\frac{1}{2}(q^2-q_0^2)$ N.	E.	S.	W.
10000	29	33	42	39	33.5	—141	—17	+321	+200
9000	28	31	42	39	32.5	—136	—47	+354	+233
8000	26	29	41	38	31.1	—146	—63	+357	+239
7000	24	27	39	36	29.0	—133	—56	+340	+228
6008	21	25	35	33	26.5	—231	—39	+262	+194
5000	20	23	32	28	23.9	— 86	—21	+227	+107
4000	18	22	30	26	22.4	— 89	— 9	+199	+ 87
3000	17	21	28	25	21.0	— 76	0	+172	+ 92
2000	14	17	24	20	17.3	— 52	— 5	+139	+ 51
1000	11	12	19	15	13.1	— 26	—14	+ 95	+ 27
000	6	7	9	9	7.3	— 9	— 2	+ 14	+ 14

Distribution of the heat, $Q - Q_0$, and the pressure, $B - B_0$.

TABLE 37—WINTER HIGH AREAS.

Height in meters.	$Q-Q_0$ N.	E.	S.	W.	$B-B_0$ N.	E.	S.	W.
10000	−11.2	+ 21.7	− 3.3	− 8.0	−10.1	− 18.2	+ 5.5	+ 19.5
9000	−13.2	+ 11.7	− 9.8	− 15.8	− 5.7	− 22.8	+ 9.6	+ 23.4
8000	−14.9	+ 13.0	− 6.4	− 19.7	− 1.1	− 30.1	+ 14.7	+ 32.1
7000	−16.2	+ 16.3	+ 0.1	− 22.9	+ 7.1	− 39.5	+ 19.2	+ 49.2
6000	−19.0	+ 15.1	+ 21.5	− 26.2	+17.5	− 51.7	+ 17.8	+ 71.7
5000	−26.3	+ 3.1	+ 71.9	− 24.9	+34.5	− 73.9	+ 1.3	+104.8
4000	−10.7	− 13.9	+ 73.9	+ 23.0	+60.1	− 75.4	− 23.6	+128.3
3000	+25.7	− 32.1	+106.0	+ 51.4	+69.1	− 94.8	− 30.6	+129.8
2000	+56.8	− 41.7	+ 59.0	+ 77.9	+83.2	−140.0	−102.1	+176.9
1000	+82.5	− 76.0	+ 38.1	+122.5	+58.1	−112.7	− 91.4	+137.2
000	+94.4	−101.6	+ 10.3	+142.5	+15.9	− 57.1	− 92.9	+ 55.5

TABLE 38.—WINTER LOW AREAS.

10000	− 1.6	− 6.5	+ 1.7	+ 7.2	+ 21.8	+12.0	−19.1	− 25.6
9000	+10.5	− 1.8	+ 7.1	+ 7.1	+ 19.9	+29.3	−21.3	− 26.3
8000	+ 9.8	+ 6.6	+ 5.5	+ 6.2	+ 15.8	+29.0	−26.4	− 33.4
7000	+ 3.6	+ 9.2	+ 3.8	+ 6.0	+ 15.2	+30.6	−31.1	− 40.2
6000	− 4.7	+ 8.2	0.0	+ 5.5	+ 16.0	+30.9	−37.9	− 49.8
5000	−16.4	+ 3.9	− 5.5	− 5.8	+ 21.6	+35.1	−46.9	− 57.3
4000	−28.2	− 7.9	− 16.6	− 9.1	+ 39.9	+39.2	−50.6	− 70.7
3000	−39.1	−19.9	− 57.7	−34.8	+ 57.9	+35.9	−56.9	− 81.6
2000	+53.9	− 5.6	−137.5	−62.7	+102.7	+53.9	−51.9	−113.3
1000	+72.8	− 7.9	−161.2	−70.6	+ 67.2	+51.0	+41.9	− 81.4
000	+36.8	− 5.8	−121.5	−54.3	+ 25.2	+30.4	+80.7	− 46.0

TABLE 39.—SUMMER HIGH AREAS.

10000	−16.7	+15.2	− 9.3	−14.8	−10.8	−12.2	+ 2.4	+ 9.4
9000	−19.5	+13.1	− 9.9	− 7.1	− 6.0	−17.9	+ 6.3	+14.5
8000	−17.8	+12.9	−11.8	− 5.0	− 076	−25.7	+11.8	+20.0
7000	−20.6	+17.3	− 0.3	− 9.9	+ 8.5	−37.3	+20.8	+26.9
6000	−26.7	+ 5.6	+35.4	+ 6.5	+18.9	−43.8	+17.9	+34.1
5000	−23.8	− 2.8	+44.3	+47.6	+26.6	−43.6	− 0.3	+40.9
4000	−31.9	− 9.0	+ 2.1	+10.4	+33.1	−45.0	− 6.6	+41.3
3000	+34.5	−25.0	−13.1	+14.2	+34.6	−51.2	− 6.4	+37.5
2000	+39.2	−30.4	+ 6.7	+16.9	+32.2	−49.3	− 8.4	+37.8
1000	+25.1	−19.2	+36.5	+25.7	+12.7	−48.2	−30.3	+26.5
000	+16.4	− 7.2	+55.0	+36.2	+ 4.5	−36.1	−44.9	+ 9.2

TABLE 40.—SUMMER LOW AREAS.

10000	+13.1	− 0.4	+ 8.8	+ 4.1	+45.1	+11.2	−18.2	−21.9
9000	+ 8.0	+ 1.6	+ 8.3	+ 5.3	+45.2	+11.3	−20.9	−25.8
8000	+ 5.8	+ 0.3	+11.2	+ 4.2	+64.9	+11.1	−29.3	−31.8
7000	+ 2.8	− 3.3	+ 8.2	+ 6.1	+57.7	+16.0	−38.8	−41.1
6000	+14.0	− 7.8	− 9.2	−17.5	+58.2	+19.8	−41.2	−47.3
5000	+20.4	− 5.7	−26.3	−23.8	+48.4	+20.0	−38.1	−40.4
4000	+18.5	+23.9	−32.3	−11.0	+43.2	+34.1	−29.6	−37.1
3000	+ 6.5	+33.9	−53.8	+ 2.8	+45.1	+ 5.5	−19.2	−46.6
2000	+38.6	− 2.3	−50.2	−19.2	+45.1	+ 6.0	− 1.1	−54.1
1000	+36.5	−13.2	−60.9	−29.9	+31.9	+10.6	+25.5	−56.0
000	+15.7	−15.4	−69.1	−31.4	+19.1	+18.4	+64.9	−34.1

is larger than n_0 and $n - n_0$ is positive, it follows that the temperature gradient is smaller for n than for n_0, and if n is smaller than n_0 the temperature gradient is larger in the disturbed region than in the normal undisturbed stratum. An examination of the tables and the fig. 14 shows that while the distribution of $n - n_0$ is similar to that of $T - T_0$ as to the sectors, yet there is a *distinct reversal between the lower and the higher strata,* occurring near the 4000-meter level. The positive (+) values of $n - n_0$ in the lower strata, showing a decrease of gradient of temperature or an inflow of heat on the western side of the high areas, become negative (−) values in the upper strata, where they indicate a more rapid loss of the temperature. The reverse conditions hold on the western half of the low areas from the surface to 10,000 meters. Hence an inflow of warm air in the lower strata, that is the southerly current, diminishes the normal gradient of temperature and produces $n - n_0$ which is a (+) quantity; similarly, an inflow of cold air in the higher strata, that is of cold air from the north, diminishes the normal temperature gradient and produces a value of $n - n_0$ which is also positive. These conditions are therefore in harmony with the observed temperature distribution in cyclones and anticyclones. The reversal between the lower and higher strata of the same sectors indicates the thermodynamic effect of the air masses striving to return to an equilibrium by reversing the gradients above and below the level of 4000 meters.

VALUES OF THE TERMS $-C_p\, n\, (T - T_0)$ AND $(z - z_0)$.

Having determined the distribution of $(T - T_0)$ and $(n - n_0)$ in the several strata, since all the terms of the equation, except the velocity, depend upon them, we proceed to compute these terms for the sake of ultimately finding the heat variations $Q - Q_0$, and the barometric pressure variations $B - B_0$. The first condition to be found is $g\,(z - z_0) = -C_p\, n_0\, (T - T_0)$ which represents the available potential energy of the air mass at a given level, to be expended in producing the kinetic energy found in cyclonic and anticyclonic circulations. Tables 29, 30, 31, 32, and fig. 15 contain the several terms. $-C_p\, n_0\, (T - T_0)$ is the potential energy, due to the fact that the mass of temperature T on the level where T_0 is the normal temperature can be reduced to equilibrium by rising or falling through a given height under the gravity acceleration g, where $g\,(z - z_0)$ is the work to be expended in falling or rising through the height $(z - z_0)$. The value of g is given on each 1000-meter level. The height $(z - z_0)$ is computed from the formula, and by fig. 15 one can see that it has the same distribution as $(T - T_0)$, upon which it depends. Thus, the positive (+) sign for $(z - z_0)$ indicates that the air mass is too cold for its level and that it can fall through a given height before reaching the normal temperature of its stratum; similarly, the negative (−) sign for $(z - z_0)$ shows that the air mass is too warm for its level and can rise through a given height to reach equilibrium. The cold sectors have the (+) sign, and the warm sectors the (−) sign, and hence the entire cold column is able to fall and the entire warm column can rise through $(z - z_0)$ meters under the influence of gravity. This is the primary source of the energy of motion in cyclones and anticyclones, this potential energy being converted into pressure differences and motions.

An inspection of the tables and the fig. 15 shows that the maximum potential energy is on the east and west sectors, at the boundary of the high and low pressure areas. Hence, there is a gradient of potential within the cold areas from the north toward the southeast or south, and within the warm areas from the south toward the northeast and north at all the levels except that next the surface. The difference between the lowest level and those above it must represent a reaction from the ground, and an accumulation of the dynamic effects from the other forces yet to be considered, such as those from the horizontal components, the deflecting force, the friction, and the other dynamic effects. Primarily, we must recognize that we deal here with a couple, one branch from the north on the west of the cyclone, and the other from the south on the east of the cyclone. The complex interactions which occur in consequence of these dispositions of warm and cold air masses form an important and difficult subject of study in hydrodynamics, which must be considered in a later paper.

DISTRIBUTION OF THE VELOCITIES q, q_0, $\frac{1}{2}(q^2-q_0^2)$.

Referring to the figs. 6 and 7, MONTHLY WEATHER REVIEW, March, 1902, for the vectors in cyclones and anticyclones, which were derived from the cloud observations made by the Weather Bureau in 1896-1897, I have transferred the values for the several gradients to Tables 33, 34, 35, and 36 in the first section q; the mean value of the q-velocity in meters per second in the eight sectors of the corresponding high and low areas gives the mean value q_0; from these are computed $\frac{1}{2}(q^2-q_0^2)$ which are plotted on fig. 16. The distribution shows that there is a prevailing northwest cold current between the high and low areas, and a southeast warm current between the low and high areas. The maximum values of $\frac{1}{2}(q^2-q_0^2)$ occur in the strata that are elevated 6000-8000 meters above the surface, and the values are somewhat greater in the cold than in the warm areas. Attention is called to the small number of units of force which are actually expended in changing the prevailing velocities of the eastward drift, in comparison with the energy available in the potential of the preceding terms.

THE DISTRIBUTION OF THE HEAT, $Q-Q_0$.

The heat term is computed from the formula,

$$Q-Q_0=\tfrac{1}{2}(q^2+q_0^2)+C_pT_0 \log T_0 (n-n_0),$$

using the values given in the preceding tables, first in mechanical units, which are then converted into calories by the divisor 4185.57. The results are tabulated in Tables 37, 38, 39, and 40, and the distribution is shown in fig. 17, which is quite similar to that for $(n-n_0)$ of fig. 14, since the term $Q-Q_0$ depends chiefly upon $(n-n_0)$. There is, therefore, the same reversal noted as in the previous case, the positive sign (+) denoting a potential energy to be turned into heat and the negative sign (—) an amount of energy to lose by cooling. One can not but be impressed with the very large amount of heat energy here available, in comparison with which the kinetic energy of motion is insignificant.

This may be the proper place to speak of the *efficiency of the atmosphere as a thermal engine*. By analogy, the warm masses are in the boiler and the cold masses are in the condenser. The percentage of heat actually turned into kinetic energy seems to be very small and the efficiency is not large. This probably comes from the general circumstance that air masses can lose their thermal contents only by action on their edges or surfaces, the interior of each mass holding its individuality for a long time under the forces which tend to destroy it by working slowly into its interior. The efficiency of the warm and cold masses under atmospheric conditions is evidently a subject which will demand very careful consideration before it can be fully analyzed and expressed as a mathematical function.

DISTRIBUTION OF THE PRESSURES, $B-B_0$.

We approach this term through the formula

$$P=\rho\left(\frac{P_0}{\rho_0}-\left[\tfrac{1}{2}(q^2-q_0^2)+g(z-z_0)\right]\right).$$

The term $\frac{P_0}{\rho_0}$ is computed from the data of Tables 17 and 18, because we have no way to determine the mean pressure P_0 and the mean density P_0 of the undisturbed stratum, except by assuming the normal conditions there found, and supposing that the local variations in pressure should be measured from them. It was for this purpose that these tables were computed, in order that they might ultimately be used in studying the local cyclonic and anticyclonic variations. As $\frac{P_0}{\rho_0}$ can be readily computed, its values will not be introduced into this paper. Furthermore, it is necessary to compute the local density, ρ, by the formula

$$\log \rho=\log \rho_0+\frac{n}{k-1}(\log T-\log T_0)$$

the n being taken as the mean n of Table 16. These values are also omitted because of the magnitude of the tables that would be required to reproduce them in this place. We finally obtain the values $P-P_0$, the variation of the pressure in mechanical units, which may be converted into millimeters by the formula,

$$B=\frac{P}{100}\times\frac{3}{4}.$$

The resulting values, $B-B_0$, are given in the second section of Tables 37, 38, 39, and 40 and they are plotted on fig. 18. The distribution is again such as has been made familiar in the preceding figures of these papers.

These pressure differences are given in millimeters, and they represent a potential energy which can be converted into cyclonic and anticyclonic motions and pressures. The fact that the observed pressures are not so great as those here given shows that the efficiency of the kinetic structure is not so great as the potential energy would indicate. Not all the available energy goes into storms, a portion being carried along in the circulating structures without transformation and a part being frittered away in internal work agitations. We have, however, shown that there seems to be an abundant supply of energy in warm and cold masses of different temperatures in the neighborhood of each other, sufficient to account for all the phenomena observed by meteorologists. It has been proven that the primary distribution is asymmetrical in respect to the centers of the low and high pressure areas. It is now one of the difficult problems to show, mathematically, how the action of these cold and warm masses, arranged as couples between the dynamic centers, is transformed from the thermodynamic structures here indicated into the hydrodynamic structures actually existing in the atmosphere. It should be remembered that the closed isobars of the lower strata, practically symmetrical about the high and low centers, are quickly modified above the surface into loops wherein the distribution of the pressure is entirely different from that at the surface as shown on the sea-level weather charts. The construction of daily and monthly isobars on the 3500-foot plane and the 10,000-foot plane for the United States during the year 1903 made this change of the structure of the isobars familiar to me. It is next in order to discuss the equations in the horizontal plane, namely,

$$q\frac{dq}{d\sigma}=\frac{dQ}{d\sigma}-c_p n_0\frac{dT}{d\sigma}-c_p T_0 \log T_0\frac{dn}{d\sigma}$$

where σ is a line in the plane dx, dy; together with the deflecting forces represented by the terms,

$$-\cos\theta(2\omega+\nu)\,v\partial x+\cos\theta(2\omega+\nu)\,u\partial y,$$

which are equal to zero, so far as the circulation is concerned, since $v\partial x=u\partial y$, though they have a decided effect upon the position of the resulting isobars; and, finally, the unknown terms representing the secondary or vortical motions induced by the dynamic motions in the sensitive hydrodynamic medium of the atmosphere.

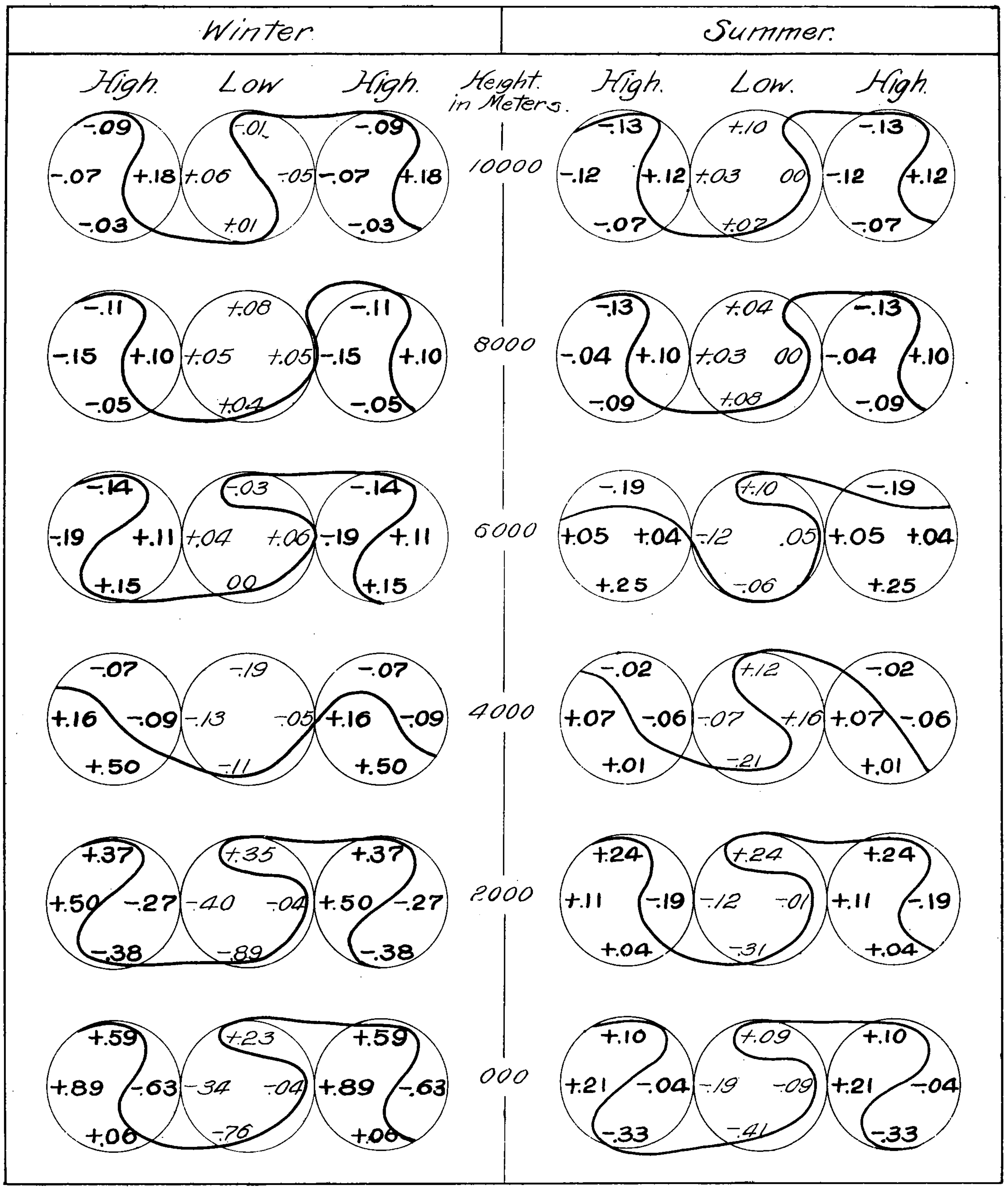

FIG. 14.—Distribution of the values of $n - n_0$ in the high and low areas.

FIG. 15.—Distribution of $z - z_0 = -\frac{C_p n_0}{g}(T - T_0)$.

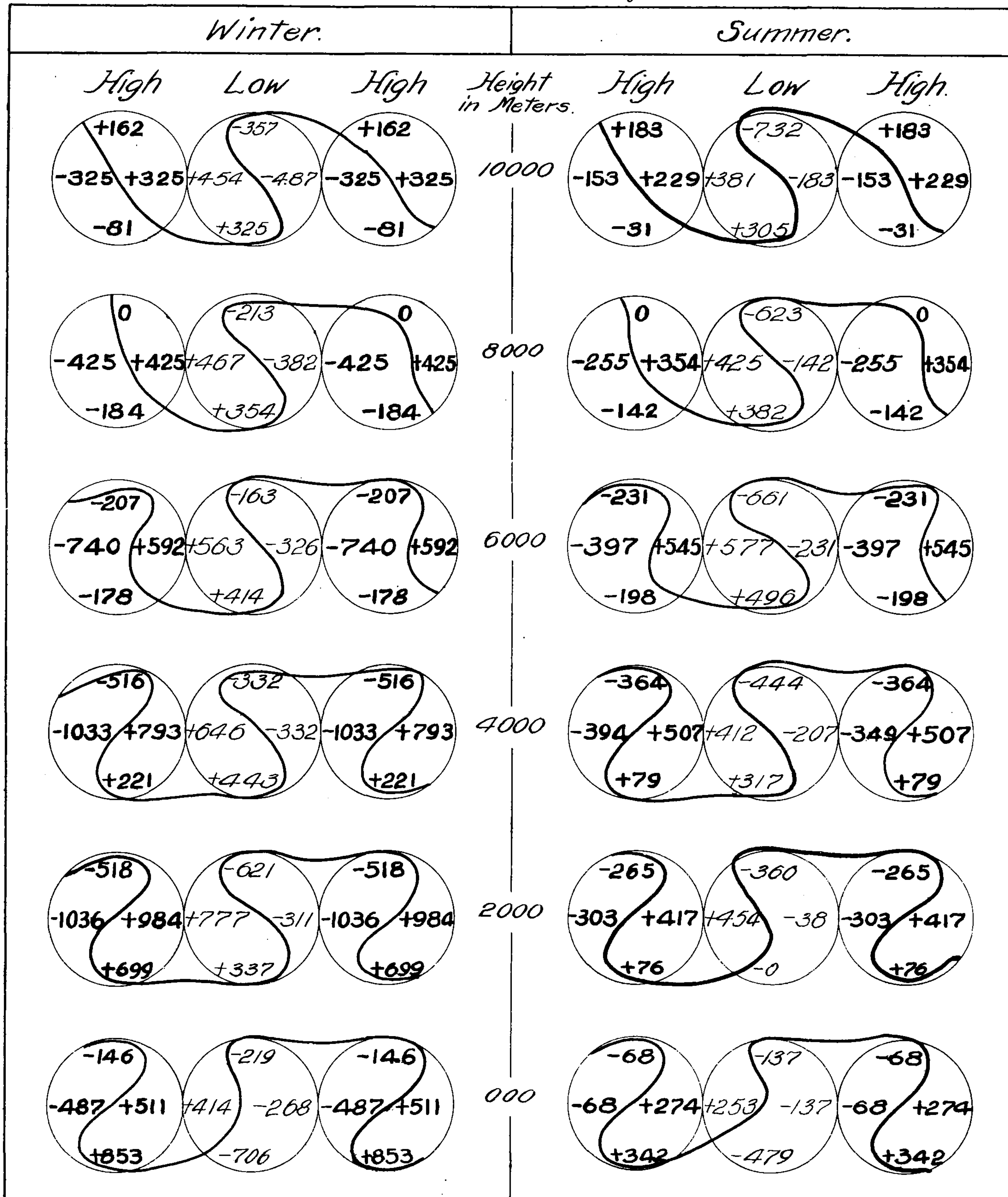

FIG. 16.—Distribution of the velocity term $\frac{1}{2}(q^2 - q_0^2)$.

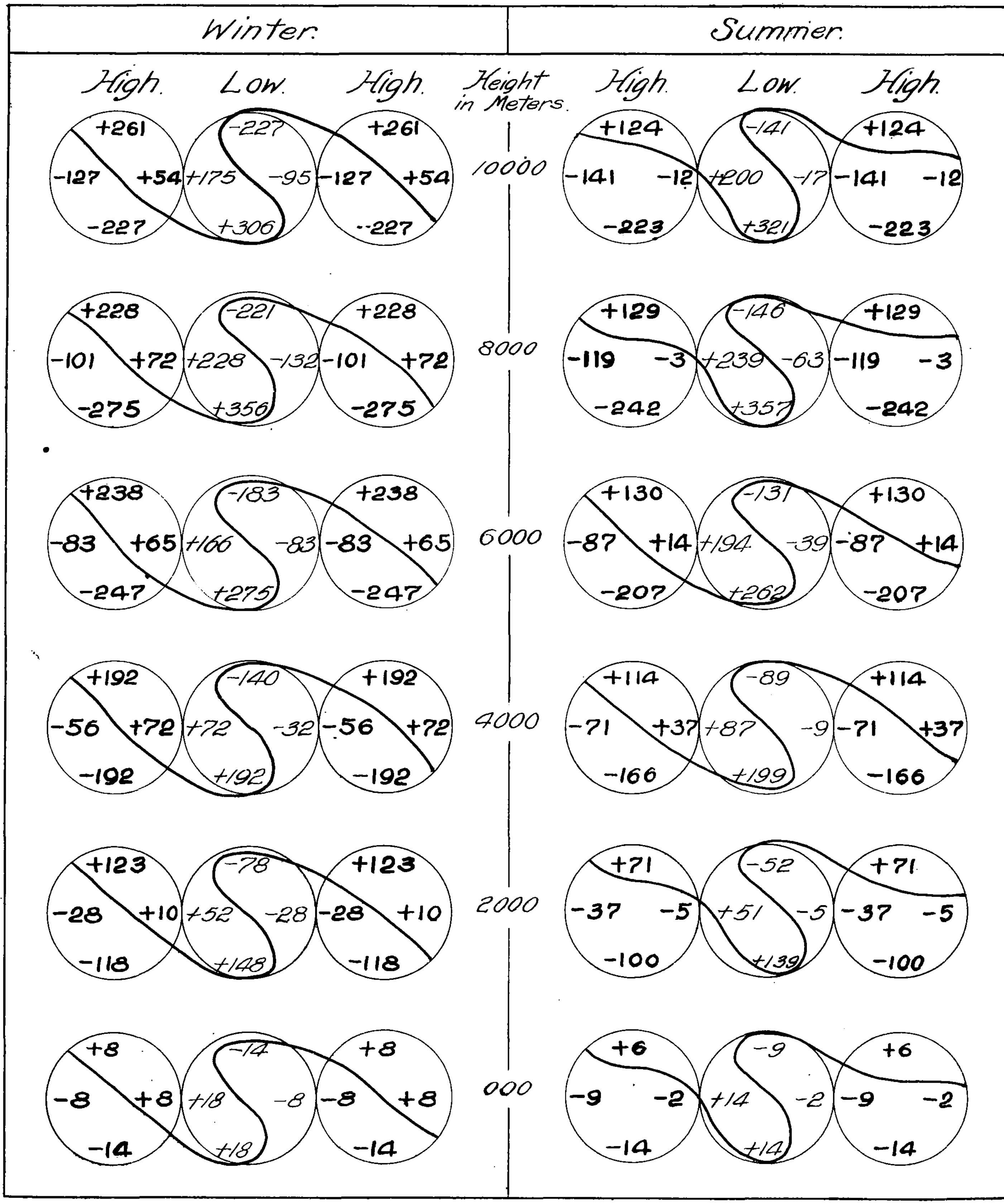

FIG. 17.—Distribution of the heat, $Q - Q_0$.